SUMMARY OF FACTORING TECHNIQUES
FALL 2007

Type (I): Factoring out the Greatest Common Factor

How do we do this? Remember the three steps:

(1) Factor out the GCF
(2) Divide each term by the GCF
(3) Simplify

Type (II): Factoring the difference of two perfect squares

How do we do this? Remember these steps:

(1) Make sure we can apply the formula: $a^2 - b^2 = (a+b)(a-b)$
(2) Apply the formula

The following chart may help you:

Squares		Cubes (uh-oh!)	
1^2	1	1^3	
2^2	4	2^3	
3^2	9	3^3	
4^2	16	4^3	
5^2	25	5^3	
6^2	36	6^3	
7^2	49	7^3	
8^2	64	8^3	
9^2	81	9^3	
10^2	100	10^3	

Also recall, zero is neither a positive nor a negative number.

Type (III): Factoring Perfect Square Trinomials

What do we look for when we have this type?

(1) First and last terms are perfect squares and positive.
(2) Middle term is even.

The formula for Type III problems is: $a^2 + 2ab + b^2 = (a+b)^2$

Type IVA: Factoring Four-term Polynomials by GROUPING

How do we do this? Remember these steps:

(1) Rearrange the terms:
ax + b + a + bx = ax + a + bx + b.

(2) Factor out an a in the first two terms, and a b in the second two terms:
ax + a + bx + b = a(x + 1) + b(x + 1)

(3) Make sure what is in the two sets of parentheses is the same:
(x + 1) = (x + 1)? YES!

(4) Now multiply what is in the parentheses by everything that is not in the parentheses:
a(x + 1) + b(x + 1) = (x + 1)(a + b)

Type IVB: Factoring Four-term Polynomials by GROUPING

How do we do this? Remember these steps:

(1) In the problem, $a^2 + 2ab + b^2 - x^2$, recognize that $a^2 + 2ab + b^2 = (a+b)^2$.

(2) Rewrite $a^2 + 2ab + b^2 - x^2$ as $(a+b)^2 - x^2$.

(3) Now $(a+b)^2 - x^2$ is a difference of two squares, so
$(a+b)^2 - x^2 = (a+b+x)(a+b-x)$.

Type V: Factoring Imperfect Square Trinomials of the Form $x^2 + bx + c$

How do we do this? Remember these steps:

(1) Make sure the coefficient in front of the x^2 is positive one.

(2) Use the following chart to determine the signs that you will use:

Sign of B	Sign of C	Factored Signs
+	+	(number + number)(number + number)
-	-	(number – large number)(number + number)
+	-	(number + large number)(number – number)
-	+	(number – number)(number – number)

(3) Find two numbers that when you multiply them, the result will be c, and when you add them, the result will be b.

Type VI: Factoring Imperfect Square Trinomials of the Form $ax^2 + bx + c$

How do we do this? We will use the Key Number Method, which has the following steps:

(1) Find the product A x C.

(2) Find two numbers E and F, such that EF = A x C and E + F = B.

(3) Rewrite $ax^2 + bx + c$ as $ax^2 + Ex + Fx + c$.
DO NOT COMBINE LIKE TERMS, it defeats the purpose!

(4) Factor.

An Example of a Type VI Problem:

Factor: $15b^2 + 4b - 4$. Note: A = 15, B = 4, and C = -4.

(1) A x C = 15 x –4 = -60

(2) EF = - 60 and E + F = 4.
The numbers whose product is – 60 and sum is 4 are: E = 10 and F = -6.

(3) Rewrite $15b^2 + 4b - 4$ as $15b^2 + 10b - 6b - 4$.

(4) To factor $15b^2 + 10b - 6b - 4$, we factor a 5b out of the first two terms, and a –2 out of the last two terms: $5b(3b + 2) - 2(3b + 2)$.

Now we make sure what is in the parentheses is the same: (3b + 2) = (3b + 2). YES!
Then we multiply what is in the parentheses by everything that is not in the parentheses:
$5b(3b + 2) - 2(3b + 2) = (3b + 2)(5b - 2)$.

Type (VII): Factoring a Perfect Cube Binomial

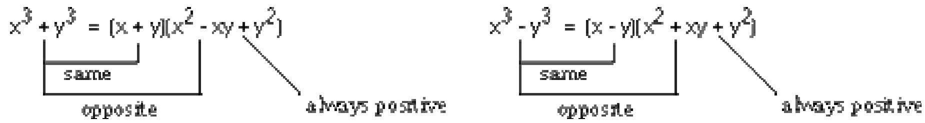
How do we do this? Remember the following steps:

(1) Make sure we can apply one of the following formulas:

Formula for factoring the sum of two cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$.
Formula for factoring the difference of two cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

NOTE: You must memorize these two formulas for the test. Remember S.O.A.P. (Same Sign, Opposite Sign, Always Positive) to help you remember these formulas.

(2) Apply the appropriate formula.



Prime or Not Prime?

We also discussed what to do if an expression cannot be factored. Expressions that cannot be factored are <u>prime</u>. We can easily find out if an expression is prime by using discriminant: $B^2 - 4AC$.

If $B^2 - 4AC$ is a perfect square, then an expression is <u>not prime</u>.
If $B^2 - 4AC$ <u>is not</u> a perfect square, then an expression is prime.

See your notes or page 456 for more information.

<u>Suggested Problems for Section 7.4 (will not be collected)</u>: 5, 6, 7, 8, 9, 10, 26, 27, 29, 30, 31, 35, 36, 38, 39, 48, 49, 51, 53, 55, 56, 57, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 79, 83, 84, 85, 89, 95, 97, 98, 99, 103, 107, 114, 115, 119, 120, 121, 131, 132, 134, 135, 137, 139